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RATE OF SEDIMENTATION OF FLOCCULATED PARTICLES

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Since 1851, when Stokes [18] put forth his equation describing the fall of a sphere in a fluid medium, work on sedimentation has progressed in several directions. Physical chemists have worked on the problem and its relation to colloid chemistry. Most of their work has been done on dispersed particles at rather low concentrations, and in small sized containers. Mining engineers have worked on it in connection with ore slimes. Their interests are more allied to those of the chemical engineer, and usually the work has been done with flocculated material on a larger scale. Robinson [15] has offered an equation covering the rate of fall of a suspension, but it has been found limited to a suspension of high concentration. No formula is available which, given the initial height and concentration of a suspension of flocculated particles and the viscosity of the medium, will allow the prediction of the height of the suspension at any subsequent time. The development of such a formulation was the purpose of the present work.

Previous Work

Stokes' Equation for the viscous fall of a small sphere in a fluid is (in consistent units)

$$\frac{H}{\theta} = \frac{2r^2(\sigma - \rho)g}{9Z} \qquad (1)$$

Roberts [15] and Oseen [12] have extended this to include larger spheres and turbulent flow. Von Blum [21] has corrected Stokes' equation for a film of adsorbed solvent that might surround the particle. In concentrated suspensions, where the particles interfere with each other in their fall, empirical equations have been suggested by Butaric and Roy [2] and McKendrick and Ponder.[10]

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Rallason [16] has pointed out that, in the final stages of sedimentation, the curve becomes logarithmic. Robinson, using the data of Adams and Glasson [1] writes Stokes' Law in the form:

$$\frac{dH}{d\theta} = K \frac{r^2(\sigma - \rho_s)}{Z_s} \qquad (2)$$

He gives as the reason for writing the equation in this form the fact that when a particle falls through a suspension of particles, the driving force is proportional, not to the difference in specific gravity of the solid and the fluid, but to the difference in the specific gravity of the solid and the suspension surrounding the particle; and the resistance to flow is proportional to the viscosity of the suspension rather than the viscosity of the medium. One of the authors [5] applied Robinson's equation to a number of chemical precipitates and found that its application was limited to concentrated suspensions where "free settling" did not exist.

As Turner [19] has pointed out, the basic fault of equations such as that of Robinson, is that they assume that a suspension in settling has, at all times, a uniform density throughout the height of the column. With suspensions of high initial concentration this is roughly true and the data are satisfied reasonably well. With suspensions of low initial concentration the assumption is not true and the initial period of constant rate sedimentation which is exhibited is not reproduced by the equation. The degree of flocculation was not specified in the above references.

If solid particles in suspension all bear a like charge, they repel each other and retain their individuality and are said to be dispersed.[8] If the proper electrolyte (flocculating agent) be added to the suspension, so that the charge on each particle is neutralized, the natural force of attraction between the particles causes two or more of them to aggregate into a floc. The settling problems encountered in the chemical industry usually involve flocculated suspensions.

Coe and Clevenger,[3] Deane,[4] Free and Ralston,[6] and Mischler,[11] working for the most part with ore slimes, have investigated this type of sedimentation, and have given us a clear picture of the mechanics of the process. Before sedimentation begins, the suspension must be uniformly distributed throughout the settling chamber by some mechanical means, which insures that all the flocs are broken up and each particle is independent of the next. On allowing the suspension to stand undisturbed, flocculation proceeds. It is obvious that the

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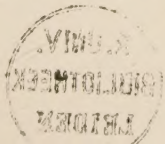
flocs, being larger in size than the single particles, will settle more rapidly than single particles, since the rate of fall is proportional to the square of the radius. For this reason, it will be noticed at the outset of sedimentation that the column falls very slowly; then, as the flocs begin to form, the rate of fall increases, until, when flocculation is complete, the rate of fall is constant. The whole process of flocculation takes place in a very short period of time in the majority of cases.

During the initial period of constant rate settling, the flocs settle more or less independently of each other with a velocity which depends, beside the variables given in Stokes' Law, upon the concentration of solids, but which is independent of the initial height of the suspension and the dimensions of the settling chamber. The flocs are loosely deposited on the bottom of the chamber, rest upon each other, and start to build a loose layer of solid. This layer of deposition is designated as the "critical settling zone" and when it reaches the top of the suspension this part of the settling curve is called the "critical settling point." Further fall of the column is then effected by "compression." During this period there is a framework of solid particles resting more or less loosely on each other, and further sedimentation takes place by water passing through the interstices between the particles. Upon further compacting of the solid, small but definite channels form in the sludge. These become larger with time, until, when sedimentation is almost complete, there appear on the surface tiny blow holes, through which the fluid can be seen to be ejected with considerable velocity.

Experimental Apparatus

The settling was done in Pyrex tubes 4.5 cms. in inside diameter and varying in effective length from 12.5 to 61.7 cms. The ends were closed with rubber stoppers, with paraffine poured in the lower end to give a uniform base. These were mounted vertically in a constant temperature water bath, 30″ high, 36″ long and 12″ thick, containing a glass plate in front for observation. The tank was equipped with a propeller for agitation and with a pair of horizontal rolls 3″ in diameter which were rotated at 100 R.P.M. The settling tubes were placed on these rolls prior to sedimentation, to allow the suspension to come to temperature and become uniformly dispersed.

Heat was supplied by a 600 watt radiant heater immersed in the water and connected by a power and telegraph relay to a mercury



thermo-regulator. Temperature control was to within 0.05° C. Temperature variation around the tubes was investigated and found to be 0.2° C. A cathatometer was mounted in front of the bath for reading suspension levels and a light was immersed in the bath to supply soft illumination when a reading was required. Otherwise, direct light was kept away from the settling tubes.

Preparation of Samples

The materials to be settled were chosen for their diverse properties. They consisted of (a) silica particles 16 mu in radius, (b) silica particles 5 mu in radius, (c) precipitated lead chromate, (d) zinc oxide as obtained commercially and (e) ferric oxide as obtained commercially.

Ground New Jersey silica was screened to minus 150 mesh and passed through a pneumatic separator built along the general lines of the air analyzer described by the U. S. Bureau of Standards.[20] The sizing of the two fractions was continued hydraulically by the method of Schramm and Scripture.[17] They use a 5% by weight suspension, but work by Roberts [13] indicates that a 10% suspension may be used without greatly widening the range of particle size included in a given cut. Dispersion of the particles was made with Philadelphia Quartz Company's "S" Brand Silicate ($Na_2O : SiO_2 = 1 : 4$) using 0.02% Na_2O based on the weight of solids in suspension as suggested by McDowell.[9]

There is no difficulty in telling when the solids are well dispersed; they settle to a hard, compact mass from which the supernatant liquid is easily poured. The charge placed on the particles by the silicate remains through four or five decantations. This indicated that it was possible to remove the charge by repeated washing when it finally became necessary to flocculate the particles. From four to six decantations were made in sizing. The diameter for the large sized silica particles was that above which flocculation was no longer exhibited. Larger particles, although they still have a charge on them, are so heavy that they will not form flocs but settle as individual particles. Particle size was determined by the sedimentation method of Schramm and Scripture. Since the particles were not spheres, the size was checked by throwing the microscopic image on a screen and making a count. Assuming that the ratio of the dimensions of cement particles applied to silica particles [20] the average radius of the silica was calculated. These values are represented by circles on the size distribu-

tion curves shown in Fig. 1, and check fairly well the values found by sedimentation.

The silica particles were washed with dilute hydrochloric acid, followed by water and then flocculated with aluminum chloride. The quantity of aluminum chloride used was that which gave the greatest rate of sedimentation. The amount was found to be 0.083 grams per liter for the smaller particles and 0.0060 grams per liter for the larger ones. It was found possible to reproduce the settling rates after a hydrochloric acid wash and reflocculation. The lead chromate was

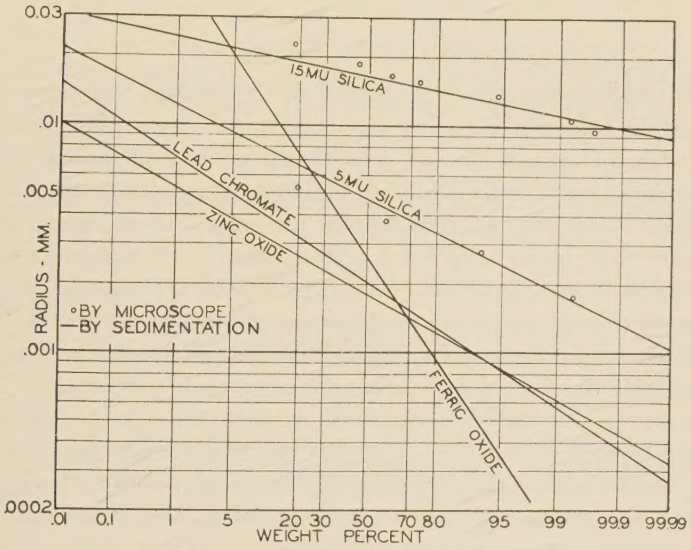


FIG. 1. Particle Size Distribution.

made by precipitation from lead acetate and sodium dichromate. Since it was impossible to duplicate the precipitate made [7] no standard procedure of precipitation was followed, but sufficient material was prepared as a stock suspension to care for all requirements. The material was washed repeatedly with water, and it was found that no flocculating agent was necessary. The zinc oxide and ferric oxide were the ordinary commercial chemicals. They were dispersed by rubbing out the solid with gum arabic solution, roughly sized to remove coarse material, washed with water and found to require no flocculating agent. The densities of the solids used were determined by the pycnometer method. The properties of the materials used are given in Table I.

TABLE I.—PROPERTIES OF MATERIALS

Material	5 *mu* Silica	16 *mu* Silica	Ferric Oxide	Zinc Oxide	Lead Chromate
Density.....	2.65	2.65	2.84	4.59	4.89
Ave. Radius, *mu*	5.13	16.7	4.93	1.97	2.27
V_u.....	0.319	0.377	0.211	0.0234	0.00986
K_c.....	—1.032	—0.49	—0.2085	—0.24	—0.274

EXPERIMENTAL PROCEDURE

The stock suspension was diluted to the desired degree and the concentration determined using a pycnometer. The settling tube was filled with the suspension and then placed under a slight vacuum to remove any air present. The cork was then placed in the top making sure that no air bubbles remained in the tube. In all cases, it was necessary to keep the suspension well agitated at all times to prevent any change in concentration. The tube was then placed on the two horizontal rolls in the constant temperature bath and allowed to come to temperature. These rolls also served the purpose of keeping the material well distributed with minimum effect on floc structure.

Upon reaching the temperature of the bath, the tube was placed in the vertical rack in the bath. It is important that the tube be absolutely vertical, since any offset will speed up the settling rate by allowing the water to rush up the high side. The height of the suspension was read by the cathatometer at about every centimeter of fall at the start and more frequently thereafter. The suspending liquid in all cases was water and temperatures of 20°, 30°, and 40° C. were used. The concentrations used were limited in the dilute range by the point at which the suspension did not settle with a sharp line. The upper limit of concentration was the point at which free settling was no longer exhibited. The initial height of the suspension was 34 cms. in most of the experiments. One set of runs was made with four different concentrations of lead chromate, using initial heights of 61.7, 48, 34 and 12.5 cms. These were run at 20° C. only. The plasticities of the suspensions were determined at the different temperatures and concentrations, but were not used in the correlation. The results of the experiments are given in Table II, and representative sedimentation curves are pictured in Figs. 2-9.

TABLE II.—SEDIMENTATION RUNS

Run No.	Material	C	Temp. °C.	Z	H_o	H_u	R_o(act)	R_o(calc)	B(act)	B(calc)
30	16muSiO_2	0.137	20	1.005	34.0	4.49	3.88	4.0	0.0653	0.0606
31	16muSiO_2	0.178	20	1.005	34.9	5.93	2.99	2.6	0.0478	0.0465
32	16muSiO_2	0.201	20	1.005	34.0	6.78	2.35	2.1	0.0427	0.0404
33	16muSiO_2	0.239	20	1.005	34.0	8.13	1.72	1.5	0.0344	0.0348
34	16muSiO_2	0.273	20	1.005	33.9	9.34	1.32	1.2	0.0309	0.0309
35	16muSiO_2	0.525	20	1.005	34.0	17.6	0.332	0.34	0.0171	0.0171
36	16muSiO_2	0.268	30	0.801	33.9	9.25	1.79	1.8	0.0399	0.0396
37	16muSiO_2	0.134	30	0.801	33.9	4.60	5.13	5.4	0.0705	0.0775
38	16muSiO_2	0.167	30	0.801	34.0	5.85	3.98	3.9	0.0611	0.0619
39	16muSiO_2	0.201	30	0.801	34.0	6.93	3.02	2.9	0.0499	0.0518
40	16muSiO_2	0.234	30	0.801	34.0	8.01	2.39	2.4	0.0411	0.0445
41	16muSiO_2	0.447	30	0.801	34.1	15.4	0.624	0.71	0.0260	0.0251
42	16muSiO_2	0.131	40	0.656	34.0	4.51	7.20	7.1	0.102	0.0983
43	16muSiO_2	0.164	40	0.656	33.8	5.50	5.50	5.6	0.0757	0.0759
44	16muSiO_2	0.179	40	0.656	33.9	6.58	4.31	4.3	0.0654	0.0645
45	16muSiO_2	0.229	40	0.656	33.9	7.69	3.45	3.4	0.0615	0.0556
46	16muSiO_2	0.262	40	0.656	33.9	8.93	2.48	2.7	0.0517	0.0491
47	16muSiO_2	0.505	40	0.656	33.8	16.6	0.598	0.90	0.0270	0.0274
61	5muSiO_2	0.474	20	1.005	33.8	17.9	0.034	0.031	.....	.....
62	5muSiO_2	0.424	20	1.005	33.9	16.1	0.046	0.041	.....	.....
63	5muSiO_2	0.351	20	1.005	34.1	13.6	0.068	0.060	.....	.....
64	5muSiO_2	0.290	20	1.005	34.1	11.2	0.098	0.091	.....	.....
65	5muSiO_2	0.236	20	1.005	34.0	9.53	0.145	0.127	.....	.....
66	5muSiO_2	0.187	20	1.005	34.1	7.58	0.235	0.225	.....	.....
67	5muSiO_2	0.156	20	1.005	34.1	6.33	0.380	0.310	.....	.....
68	5muSiO_2	0.123	20	1.005	33.9	5.06	0.565	0.500	.....	.....

TABLE II.—(*Continued*)

Run No.	Material	C	Temp. °C.	Z	H_o	H_u	R_o(act)	R_o(calc)	B(act)	B(calc)
69	5muSiO_2	0.465	30	0.801	34.1	17.7	0.048	0.054	.....	.....
70	5muSiO_2	0.352	30	0.801	34.2	13.9	0.078	0.094	.....	.....
71	5muSiO_2	0.283	30	0.801	33.9	11.1	0.131	0.150	.....	.....
72	5muSiO_2	0.233	30	0.801	34.0	9.24	0.197	0.235	.....	.....
73	5muSiO_2	0.186	30	0.801	34.0	7.50	0.344	0.36	.....	.....
74	5muSiO_2	0.151	30	0.801	34.0	6.23	0.551	0.54	.....	.....
75	5muSiO_2	0.125	30	0.801	34.0	5.53	0.769	0.690	.....	.....
76	5muSiO_2	0.454	40	0.656	34.0	18.1	0.069	0.081	.....	.....
77	5muSiO_2	0.374	40	0.656	33.9	15.3	0.100	0.116	.....	.....
78	5muSiO_2	0.308	40	0.656	33.9	12.4	0.144	0.185	.....	.....
79	5muSiO_2	0.250	40	0.656	33.9	10.3	0.224	0.280	.....	.....
80	5muSiO_2	0.205	40	0.656	34.0	8.62	0.353	0.400	.....	.....
81	5muSiO_2	0.161	40	0.656	34.0	6.76	0.640	0.670	.....	.....
115	$PbCrO_4$	0.0185	20	1.005	61.7	18.6	1.282	1.30	0.00308	0.00467
116	$PbCrO_4$	0.0185	20	1.005	48.4	15.4	1.46	1.16	0.00722	0.00596
117	$PbCrO_4$	0.0185	20	1.005	33.8	11.4	1.328	1.06	0.0103	0.0085
118	$PbCrO_4$	0.0185	20	1.005	12.5	5.4	0.63	0.67	0.0222	0.0230
119	$PbCrO_4$	0.0101	20	1.005	61.7	10.7	3.3	3.3	0.00606	0.00645
120	$PbCrO_4$	0.0101	20	1.005	48.2	9.1	3.43	2.9	0.01065	0.00825
122	$PbCrO_4$	0.0101	20	1.005	12.4	3.4	3.12	1.55	0.0442	0.0318
123	$PbCrO_4$	0.01005	20	1.005	61.7	10.5	4.8	3.4	0.00901	0.00645
124	$PbCrO_4$	0.01005	20	1.005	48.2	8.2	4.47	3.4	0.010	0.00825
125	$PbCrO_4$	0.01005	20	1.005	34.2	5.59	4.33	3.65	0.0125	0.0188
126	$PbCrO_4$	0.01005	20	1.005	12.8	3.21	2.12	1.8	0.0802	0.031
127	$PbCrO_4$	0.00792	20	1.005	61.7	7.93	7.13	5.1	0.0139	0.0095

TABLE II.—(*Continued*)

Run No.	Material	C	Temp. °C.	Z	H_o	H_u	R_o(act)	R_o(calc)	B(act)	B(calc)
128	$PbCrO_4$	0.00792	20	1.005	48.2	6.49	6.44	4.9	0.0152	0.0122
129	$PbCrO_4$	0.00792	20	1.005	34.0	4.96	5.00	4.3	0.0155	0.0173
131	$PbCrO_4$	0.0185	30	0.801	34.4	10.8	1.87	1.8	0.00959	0.0106
133	$PbCrO_4$	0.01005	30	0.801	34.4	5.59	3.93	5.0	0.0116	0.0147
134	$PbCrO_4$	0.00792	30	0.801	34.1	5.06	5.11	5.0	0.0182	0.0216
135	$PbCrO_4$	0.01850	40	0.656	34.3	11.2	2.21	2.4	0.0145	0.0130
137	$PbCrO_4$	0.01005	40	0.656	34.5	5.75	3.19	6.4	0.0132	0.0179
138	$PbCrO_4$	0.00792	40	0.656	34.1	5.08	4.64	7.2	0.0198	0.0264
139	ZnO	0.0302	20	1.005	34.0	7.88	1.37	1.19	0.00364	0.00383
140	ZnO	0.0250	20	1.005	34.1	6.72	1.53	1.60	0.00361	0.00407
141	ZnO	0.0205	20	1.005	34.2	5.27	2.18	2.4	0.00432	0.00451
142	ZnO	0.0157	20	1.005	34.2	4.89	3.56	2.75	0.00785	0.00815
143	ZnO	0.0302	30	0.801	34.2	7.79	1.67	1.8	0.00442	0.00481
144	ZnO	0.0250	30	0.801	34.1	6.77	2.17	2.3	0.00582	0.00511
145	ZnO	0.0205	30	0.801	34.3	5.01	3.26	3.55	0.00579	0.00566
146	ZnO	0.0157	30	0.801	33.9	4.56	4.16	4.3	0.0100	0.0103
147	ZnO	0.0302	40	0.656	34.0	7.73	2.29	2.6	0.00698	0.00586
148	ZnO	0.0252	40	0.656	34.1	6.64	2.50	3.3	0.00729	0.00625
149	ZnO	0.0205	40	0.656	34.5	5.09	3.43	5.0	0.00659	0.00691
150	ZnO	0.0157	40	0.656	34.0	4.28	5.29	6.1	0.0129	0.0125
154	Fe_2O_3	0.1486	20	1.005	33.9	6.21	1.37	1.14	0.00309	0.00308
155	Fe_2O_3	0.1058	20	1.005	34.6	4.0	3.15	2.56	0.00347	0.00351
156	Fe_2O_3	0.0963	20	1.005	33.9	3.95	3.11	2.6	0.0035	0.0035
157	Fe_2O_3	0.0634	20	1.005	34.1	2.97	4.69	4.0	0.00401	0.00402

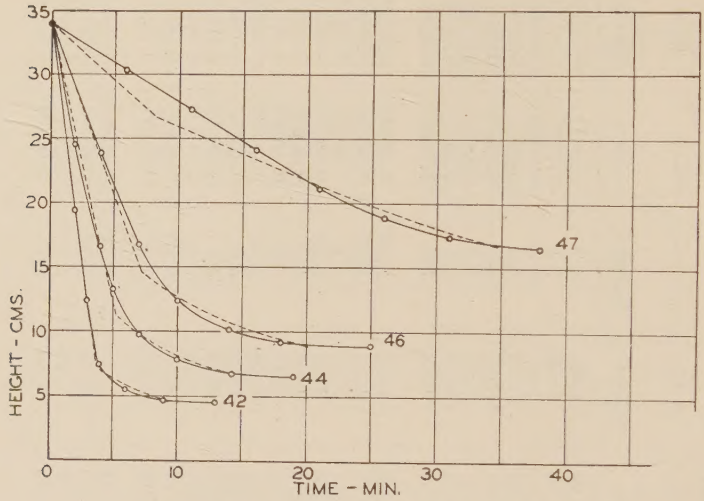


FIG. 2. Sedimentation of 16μ Silica—40° C.

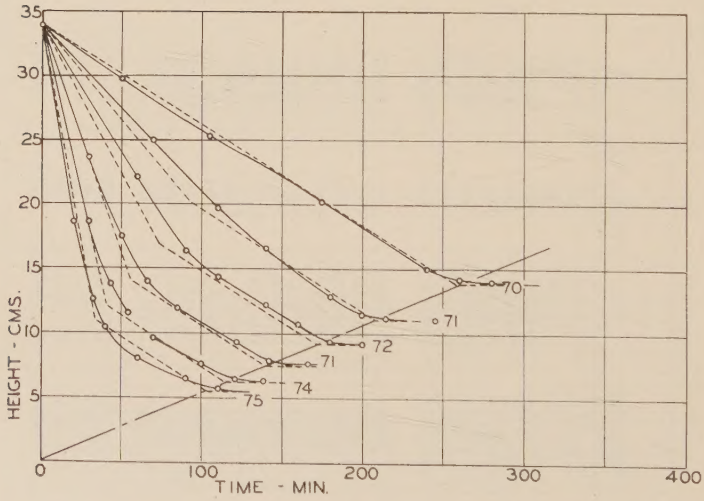


FIG. 3. Sedimentation of 5μ Silica—30° C.

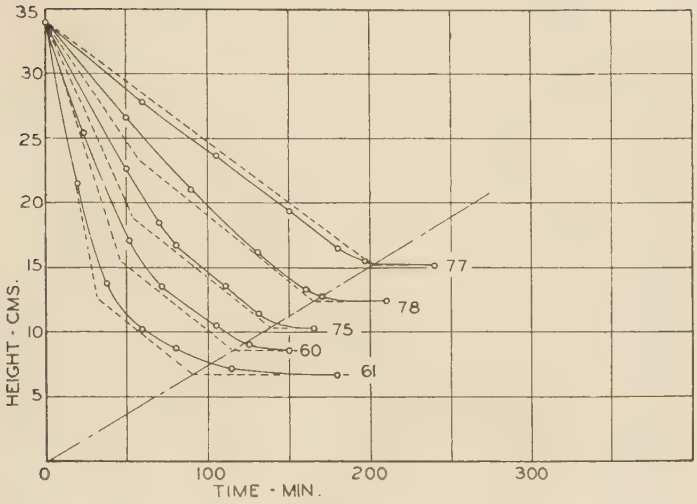


FIG. 4. Sedimentation of 5μ Silica—40° C.

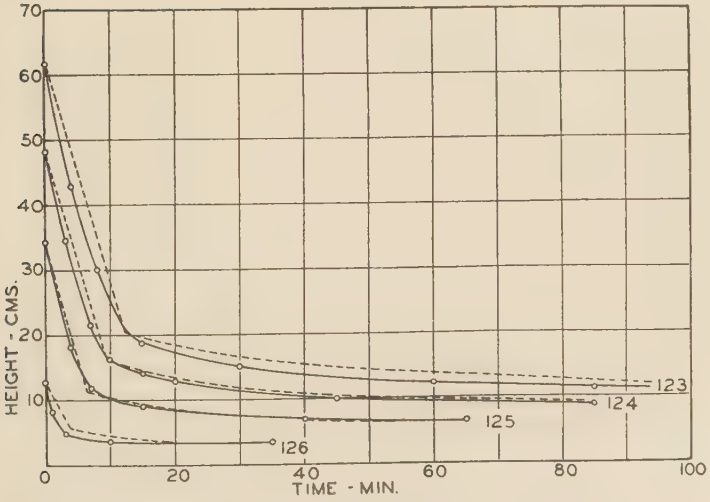


FIG. 5. Sedimentation of Lead Chromate—20° C.

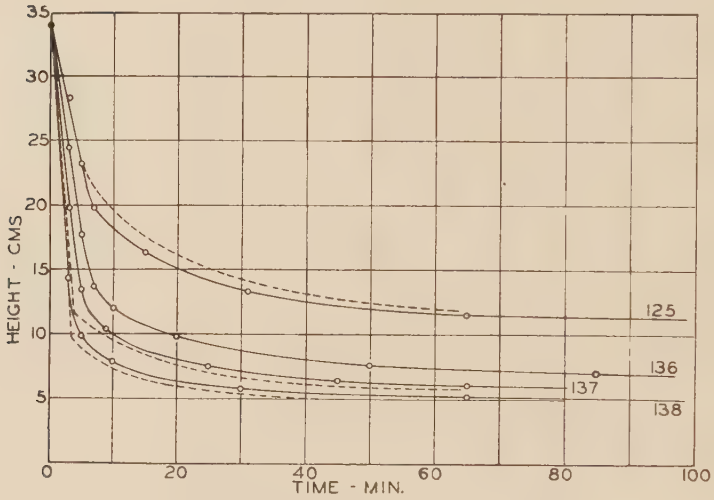


Fig. 6. Sedimentation of Lead Chromate—40° C.

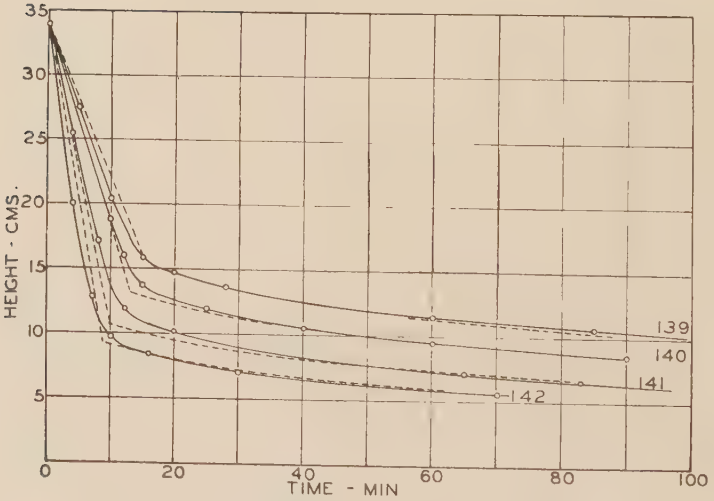


Fig. 7. Sedimentation of Zinc Oxide—20° C.

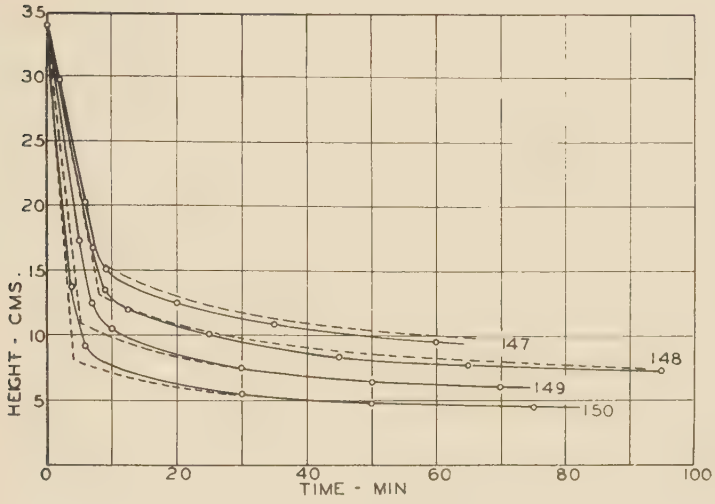


Fig. 8. Sedimentation of Zinc Oxide—40° C.

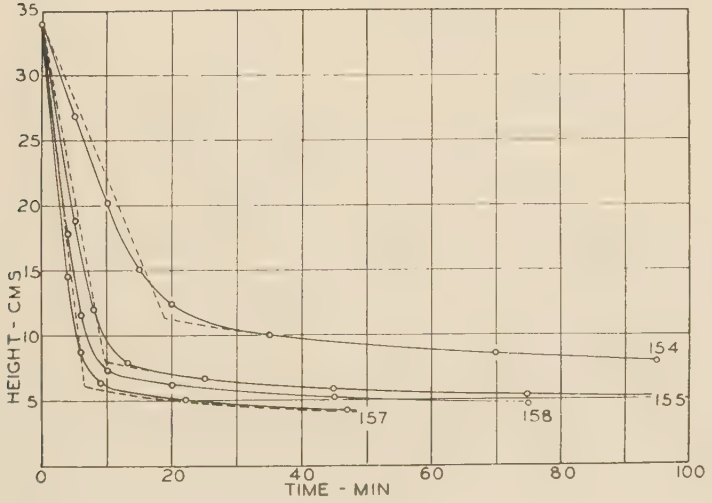


Fig. 9. Sedimentation of Ferric Oxide—20° C.

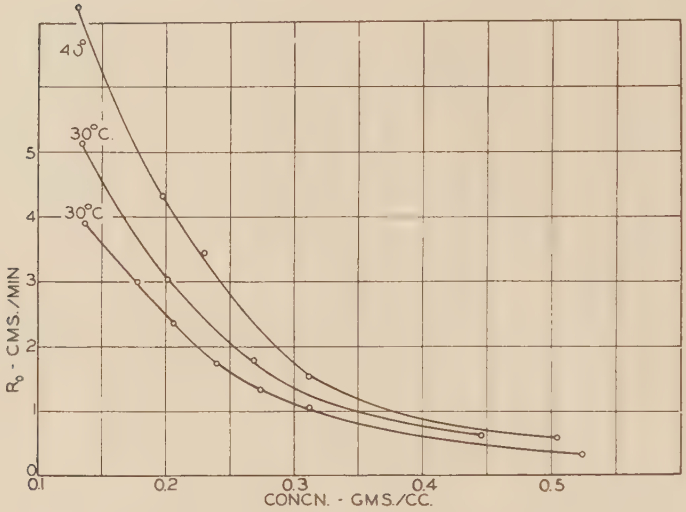


FIG. 11. Initial Settling Rates vs. Concentration. 16μ Silica.

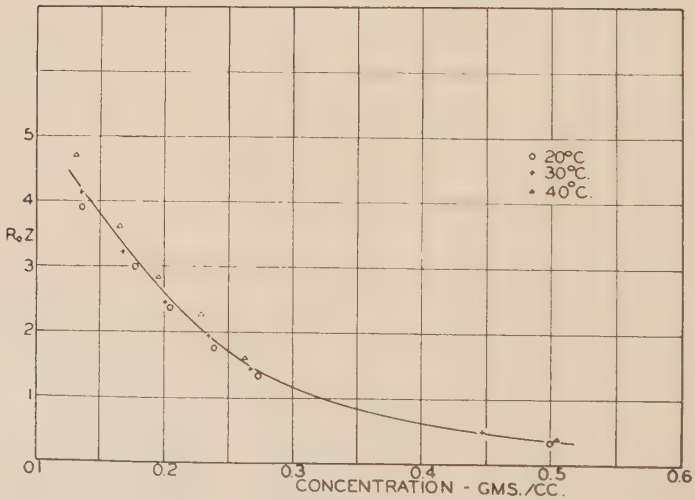


FIG. 12. (Initial Rates) (Viscosity) vs. Concentration. 16μ Silica.

The degree of these variations is a measure of the reproducibility of the results. The curves are seen to have the same general slope. For any given material, of given size, it is possible, then, if the initial rates at a few concentrations are obtained, to plot the initial rates against the corresponding concentrations, and obtain by interpolation the initial rate at any desired concentration. It should be borne in mind, however, that these curves are for materials that are fully floccu-

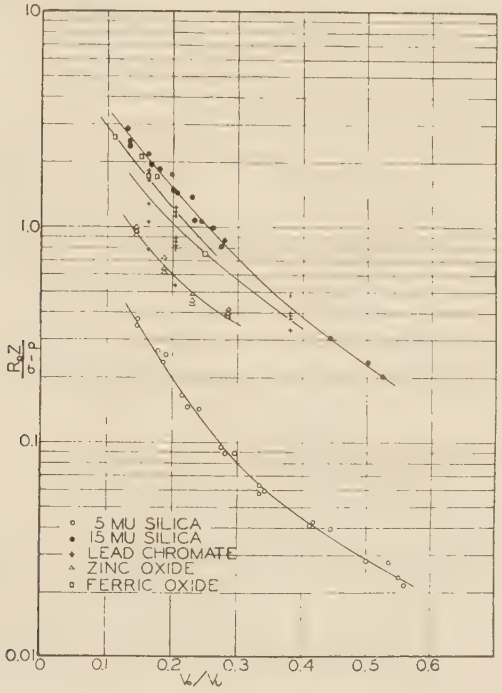


FIG. 13. Alignment of Initial Rates.

lated. If the material is only partially so, a curve with a steeper slope results, and such a plot can be used to check the completeness of flocculation.

A logical method of predicting the intial rate of settling of a floc composed of particles of known size and density would be to apply to it, Stokes' Law. This requires knowing the density and radius of the floc, and the viscosity of the medium through which it was falling. Since these variables cannot be determined directly, combinations of them were tried as correlating variables, but no dimen-

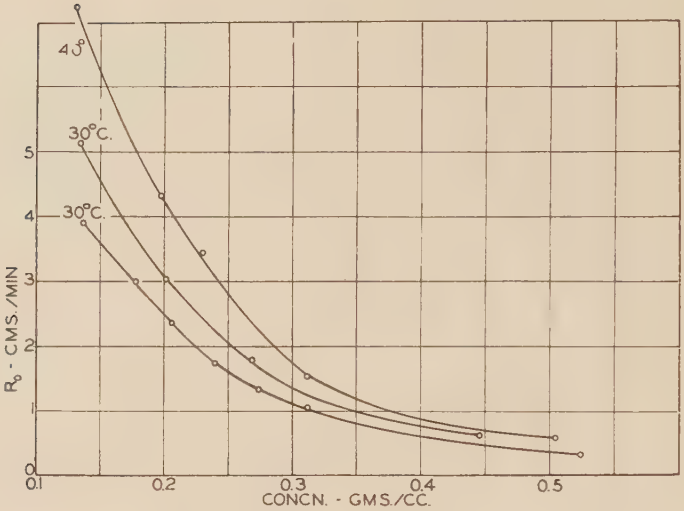


FIG. 11. Initial Settling Rates vs. Concentration. 16μ Silica.

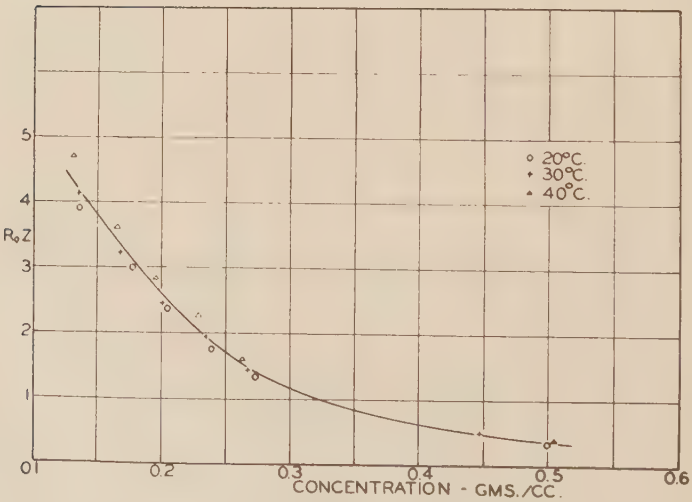


FIG. 12. (Initial Rates) (Viscosity) vs. Concentration. 16μ Silica.

The degree of these variations is a measure of the reproducibility of the results. The curves are seen to have the same general slope. For any given material, of given size, it is possible, then, if the initial rates at a few concentrations are obtained, to plot the initial rates against the corresponding concentrations, and obtain by interpolation the initial rate at any desired concentration. It should be borne in mind, however, that these curves are for materials that are fully floccu-

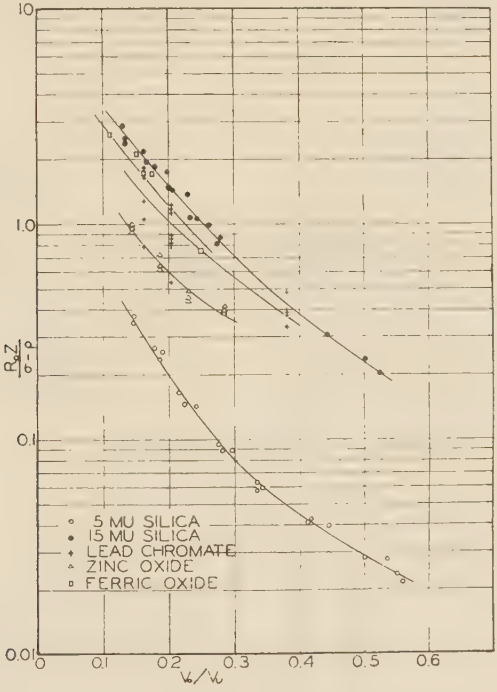


FIG. 13. Alignment of Initial Rates.

lated. If the material is only partially so, a curve with a steeper slope results, and such a plot can be used to check the completeness of flocculation.

A logical method of predicting the intial rate of settling of a floc composed of particles of known size and density would be to apply to it, Stokes' Law. This requires knowing the density and radius of the floc, and the viscosity of the medium through which it was falling. Since these variables cannot be determined directly, combinations of them were tried as correlating variables, but no dimen-

sionally sound correlation could be obtained. The next approach was to write Stokes' Law for a separate particle and multiply it by suitable non-dimensional groups to obtain a correlation for the material in the flocculated condition. After investigating several dimensionless groups, the correlation finally adopted was,

$$R_o = \phi \frac{r^2(\sigma - \rho)}{Z} \cdot \frac{H_o}{H_u} \cdot (0.415 - V_u)^{1.4} \qquad (4)$$

The relationship is plotted in Fig. 14. The predicted values of the initial rate in the seventy-six runs made were within $\pm 20\%$ of the actual values in 80% of the cases.

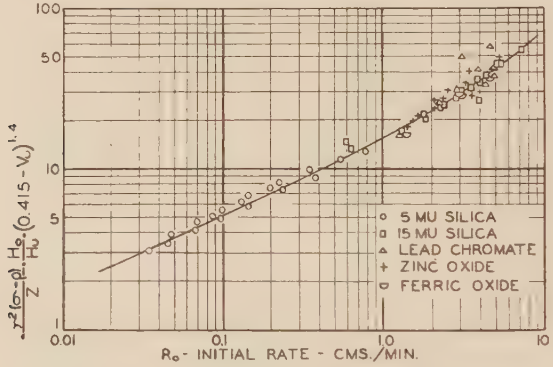


FIG. 14. Correlation of Initial Settling Rates.

This relationship is for the most part empirical, but does have some rational basis. The initial rate increases as the concentration decreases, and the ratio H_o/H_u is equivalent to the reciprocal of the concentration, since the ultimate height is practically constant. Likewise, it has been noted that the size of the floc, and therefore the initial rate, increases as the degree of flocculation increases. The quantity $(0.415 - V_u)$ is a measure of the intensity of the flocculation, since it has been observed that the more strongly the particles flocculate, the less dense is the final sediment, and, hence, the greater is $(0.415 - V_u)$.

Rollason [16] has shown that the sedimentation curve is logarithmic. It was found convenient in the present case to use the equation,

$$\log (H/H_u) = K_c \log B\theta \qquad (5)$$

The values of H/H_u were plotted against the time on log-log paper to obtain the value of K_c. It was found that K_c was substantially the

same for any given material, regardless of the initial height of the suspension, the initial concentration, or the temperature. Having the value of K_c, it was possible to take values of H and θ and obtain a value for B. This was done at the beginning and end of the compression period, and the two values averaged. Usually they varied little. The values of B, of course, are different for each curve, but it was found that the values of $(B)(H_o)(Z)$ when plotted against the concentration gave a set of points which fell in line. Using preliminary experiments to obtain such a curve, the value of B can be obtained for any set of initial conditions. The calculated values were within $\pm 20\%$ of the actual values in 80% of the runs.

From the foregoing information, the sedimentation curve can be reconstructed. The height of the column is plotted against the time. The chosen value of H_o is placed at $\theta = 0$, and a straight line drawn downward, at a slope indicated by the value of the initial rate. With the values of K_c, B and H_u known, the values of H at various values of θ are calculated and the compression part of the curve drawn in, until it intersects the initial rate curve. Since no allowance has been made for the transition part of the curve, the break at the meeting point is smoothed out with a fillet. Experience dictates the extent of this. The reconstructed curves are plotted along with the actual curves in broken lines. The accuracy is fairly good considering the reproducibility of the data.

The smaller sized silica particles were found to follow Type II in their manner of settling. The initial rate varied with the concentration and is given by Equation 4. This was followed by a second period of constant rate settling, which varied with the temperature, but was independent of the initial concentration. It was not possible to correlate this second constant rate for different temperatures by use of the viscosity. The compression and transition zones were very short in these runs, and were neglected. It was found that a straight line could be drawn through the origin and the points where the curves reached their ultimate height, and this fact was used as a basis to reconstruct the curves. Some preliminary runs are made to determine this line and the secondary constant rate period of settling. At a point where this line intersects the ultimate height, a line is projected upwards at a slope indicated by the secondary constant rate. This line is allowed to intersect the initial settling line, constructed as previously described. The method is made clear on the plots. Small fillets are inserted at the breaks in the curves.

Summary

The sedimentation of flocculated particles was studied using five different materials, (a) silica particles 16 mu in radius, (b) silica particles 5 mu in radius, (c) lead chromate, (d) zinc oxide and (e) ferric oxide. The temperatures varied from 20° to 40° C., and the initial heights of the suspensions from 12.5 to 61.7 cms. The concentrations varied from the point where the suspension no longer settled with a sharp line up to the point where free settling was no longer exhibited.

A method of reconstructing the curves is proposed which divides the process into the initial straight line or free settling period and the final or compression period. The correlation suggested for the initial rate is

$$R_o = \phi \frac{r^2(\sigma - \rho)}{Z} \cdot \frac{H_o}{H_u} \cdot (0.415 - V_u)^{1.4}$$

and that for the compression period

$$\log \frac{H}{H_u} = K_c \log(B\theta)$$

The general accuracy of the reconstruction is ±20% in 80% of the cases.

Nomenclature

a, b = Constants.
H = Height of suspension at time θ — centimeters.
H_o = Initial height of suspension.
H_u = Ultimate height of suspension.
R_o = Initial rate of settling centimeters/minute.
θ = Time — Minutes.
r = Radius of particle in microns.
σ = Density of solid particles — grams/cc.
ρ = Density of medium — grams/cc.
Z = Viscosity of medium — centipoises.
Z_s = Viscosity of suspension — centipoises.
V = Volume fraction of solids — cc. of solids/cc. of suspension.
C = Concentration of solids, grams of solids/cc. of suspension.
D = Dilution, grams liquid/gram of solid.
K_c = Compression constant.—Equation (5).
B = Compression constant.—Equation (5).
$_o$ = Initial condition, at zero time.
$_u$ = Ultimate condition.

LITERATURE CITED

1. Adams & Glasson, M.I.T. Thesis, 1925.
2. Boutaric & Roy, *Compt. Rend.*, **190**, 272 (1930).
3. Coe and Clevenger, *Trans. Am. Inst. Mining Engrs.*, **55**, 356 (1916).
4. Deane, *Trans. Am. Electrochem. Soc.*, **37**, 71 (1920).
5. Egolf, M.I.T. Thesis, 1929.
6. Free, *J. Phys. Chem.*, **13**, 114 (1909).
7. Free and Ralston, *Eng. and Mining J.*, **101**, 249 et seq. (1916).
8. Kruyt, H. R., "Colloids," p. 67, et seq., New York, John Wiley & Sons, 1930.
9. McDowell, *J. Am. Ceram. Soc.*, **10**, 225 (1927).
10. McKendrick and Ponder, *Proc. Roy. Soc. Edinburgh*, **49**, 170 (1929).
11. Mishler, *Trans. Am. Inst. Mining Eng.*, **58**, 102 (1918).
12. Oseen, *Arkiv. f. Mat., Astrom. och Fysik*, **6**, No. 29 (1910) and **9**, No. 16 (1913).
13. Roberts, E. J., Private Communication, 1935.
14. Roberts, *Trans. Inst. Chem. Engrs.*, **11**, 124 (1933).
15. Robinson, *Ind. and Eng. Chem.*, **18**, 869 (1928).
16. Rollason, M.I.T. Thesis, 1913.
17. Schramm and Scripture, *J. Am. Ceram. Soc.*, **8**, 243 (1925).
18. Stokes, "Mathematical and Physical Papers, Vol. II, 10 and Vol. III, 56. Cambridge University Press, 1880.
19. Turner, M.I.T. Thesis, 1926 and 1928.
20. U. S. Bureau of Standards, Technical Paper No. 48.
21. Von Blum, *Farbe u. Lack*, 1927, 329.